On a

$$E\left(X \mid Y=y\right)=\int_0^y x f_{X|Y}\left(x,y\right)dx, \qquad \mathbf{(0.5)}$$

où

$$f_{X|Y}\left(x,y\right)=\frac{\frac{5}{18\sqrt{x}\sqrt{y}}}{\frac{5}{9}}=\frac{1}{2\sqrt{x}\sqrt{y}}. \qquad \mathbf{(0.5)}$$

Donc

$$E\left(X \mid Y=y\right)=\int_0^y \frac{x}{2\sqrt{x}\sqrt{y}}dx=\frac{1}{3}y. \qquad \mathbf{(0.5)}$$

4. **<u>Méthode 1.</u>** On a

$$Z=2E\left(X \mid Y\right)-3=\frac{2}{3}Y-3, \qquad \mathbf{(0.25)}$$

et

$$E\left(Z\right)=g_Z'\left(s\right)\big|_{s=0}, \qquad \mathbf{(0.5)}$$

où

$$g_Z\left(s\right)=e^{-3s}g_Y\left(\frac{2}{3}s\right) \quad \mathbf{(0.25)} \Longrightarrow g_Z'\left(s\right)=-3e^{-3s}g_Y\left(\frac{2}{3}s\right)+e^{-3s}g_Y'\left(\frac{2}{3}s\right), \qquad \mathbf{(0.5)}$$

avec

$$g_Y\left(\frac{2}{3}s\right)=\int_0^{9/5}\frac{5}{9}e^{\frac{2}{3}sy}dy=\frac{5e^{\frac{6}{5}s}-5}{6s}, \qquad \mathbf{(0.5)}$$

alors

$$g_Y'\left(\frac{2}{3}s\right)=\frac{\left(6s-5\right)e^{\frac{6}{5}s}+5}{6s^2}. \qquad \mathbf{(0.5)}$$

En utilisant la règle de l'Hôpital, on obtient

$$\begin{aligned} g_Z'\left(s\right)\big|_{s=0} &= -3e^{-3s}\,g_Y\left(\frac{2}{3}s\right)\bigg|_{s=0}+e^{-3s}\,g_Y'\left(\frac{2}{3}s\right)\bigg|_{s=0} \\ &= -3+\frac{3}{5} \qquad \mathbf{(0.5)} \\ &= -\frac{12}{5}, \qquad \mathbf{(0.25)} \end{aligned}$$

et

$$E\left(Z\right)=-\frac{12}{5}. \qquad \mathbf{(0.25)}$$

<u>Méthode 2.</u> On a

$$Z=2E\left(X \mid Y\right)-3=\frac{2}{3}Y-3=h\left(Y\right), \qquad \mathbf{(0.25)}$$

et

$$E\left(Z\right)=g_Z'\left(s\right)\big|_{s=0}, \qquad \mathbf{(0.25)}$$

où

$$g_Z\left(s\right)=\int_{-3}^{-9/5}e^{sz}f_Z\left(z\right)dz, \qquad \mathbf{(0.25)}$$

avec

$$\begin{aligned} f_Z\left(z\right) &= \left|\left(h^{-1}\left(z\right)\right)'\right| f_Y\left(h^{-1}\left(z\right)\right) \qquad \mathbf{(0.25)} \\ &= \left|\left(\frac{3z+9}{2}\right)'\right|\frac{5}{9} \qquad \mathbf{(0.25)} \\ &= \frac{5}{6}, \qquad \mathbf{(0.25)} \end{aligned}$$