Corrigé type de l'examen: Complément de la Théorie des Probabilités (2023/2024)

Exercice 1 **(14 points).**

1. On a

$$\int_0^{\frac{9}{5}}\int_x^{\frac{9}{5}} \frac{\alpha}{\sqrt{xy}}dydx = \alpha \int_0^{\frac{9}{5}} \frac{1}{\sqrt{x}}\left(\int_x^{\frac{9}{5}} \frac{1}{\sqrt{y}}dy\right)dx = 1. \quad \mathbf{(0.5)}$$

Donc $\alpha = \frac{5}{18}$ **(1)** et

$$f_{XY}(x,y) = \frac{5}{18\sqrt{xy}}\mathbf{1}_{]0,\frac{9}{5}[}(x)\,\mathbf{1}_{[x,\frac{9}{5}[}(y).$$

2. La densité marginale de X est

$$\begin{aligned} f_X(x) &= \frac{5}{18}\int_x^{\frac{9}{5}} \frac{1}{\sqrt{x}\sqrt{y}}dy \quad \mathbf{(0.5)} \\ &= \frac{\sqrt{5}}{3\sqrt{x}} - \frac{5}{9}. \quad \mathbf{(1)} \end{aligned}$$

. La densité marginale de Y est

$$\begin{aligned} f_Y(y) &= \frac{5}{18}\int_0^{y} \frac{1}{\sqrt{x}\sqrt{y}}dx \quad \mathbf{(0.5)} \\ &= \frac{5}{9}. \quad \mathbf{(0.5)} \end{aligned}$$

3. La covariance est

$$cov(X,Y) = E(XY) - E(X)E(Y). \quad \mathbf{(0.25)}$$

On a

$$\begin{aligned} E(X) &= \int_0^{\frac{9}{5}} x\left(\frac{\sqrt{5}}{3\sqrt{x}} - \frac{5}{9}\right)dx \quad \mathbf{(0.5)} \\ &= \frac{3}{10}, \quad \mathbf{(0.5)} \end{aligned}$$

$$\begin{aligned} E(Y) &= \frac{5}{9}\int_0^{\frac{9}{5}} ydy \quad \mathbf{(0.5)} \\ &= \frac{9}{10}, \quad \mathbf{(0.5)} \end{aligned}$$

et

$$\begin{aligned} E(XY) &= \frac{5}{18}\int_0^{\frac{9}{5}} \frac{x}{\sqrt{x}}\left(\int_x^{\frac{9}{5}} \frac{y}{\sqrt{y}}dy\right)dx \quad \mathbf{(0.5)} \\ &= \frac{9}{25}. \quad \mathbf{(0.5)} \end{aligned}$$

Donc

$$cov(X,Y) = \frac{9}{25} - \frac{27}{100} = \frac{9}{100}. \quad \mathbf{(0.25)}$$