minimaux : $\Phi_{II}(X) = \max(X_1, \min(X_2, X_3, X_4))$ *donc*

$$\begin{aligned} R_{II}(p) &= P(\Phi_{II}(X) = 1) = P(\max(X_1, \min(X_2, X_3, X_4)) = 1) \\ &= 1 - P(\max(X_1, \min(X_2, X_3, X_4)) = 0) = 1 - P(X_1 = 0, \min(X_2, X_3, X_4) = 0) \\ &= 1 - P(X_1 = 0) P(\min(X_2, X_3, X_4) = 0) \\ &= 1 - P(X_1 = 0)(1 - P(\min(X_2, X_3, X_4) = 1)) \\ &= 1 - P(X_1 = 0)(1 - P(X_2 = 1) P(X_3 = 1) P(X_4 = 1)) \\ &= 1 - (1 - p_1)(1 - p_2 p_3 p_4) = p_1 \coprod p_2 p_3 p_4 ....(\mathbf{0.75\ points}) \end{aligned}$$

-***Si les états des composants forment une Chaine de Markov homogène de loi initiale*** $\left(\frac{1}{2}, \frac{1}{2}\right)$ ***et de matrice de transition*** $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

c'est à dire $\left(\frac{1}{2}, \frac{1}{2}\right) = (P(X_1 = 0), P(X_1 = 1)) = (1 - p_1, p_1)$ *et* $p_{0,0} = p_{0,1} = p_{1,0} = p_{1,1} = \frac{1}{2}$***......(0.5 points)***

Dans ce cas on utilise les ensembles M *et* M' *et la propriété de Markov on obtient*

Système I :* *..................(0.75 points)***

$$\begin{aligned} R_I(p) &= P(\Phi_I(X) = 1) = P(X \in M) = P(X_1 = 1, X_2 = 1, X_3 = 1, X_4 = 1) \\ &\quad + P(X_1 = 1, X_2 = 1, X_3 = 1, X_4 = 0) + P(X_1 = 1, X_2 = 1, X_3 = 0, X_4 = 1) \\ &\quad + P(X_1 = 1, X_2 = 0, X_3 = 1, X_4 = 1) + P(X_1 = 1, X_2 = 1, X_3 = 0, X_4 = 0) \\ &\quad + P(X_1 = 1, X_2 = 0, X_3 = 1, X_4 = 0) + P(X_1 = 1, X_2 = 0, X_3 = 0, X_4 = 1) \\ &= p_1 p_{1,1} p_{1,1} p_{1,1} + p_1 p_{1,1} p_{1,1} p_{1,0} + p_1 p_{1,1} p_{1,0} p_{0,1} \\ &\quad + p_1 p_{1,0} p_{0,1} p_{1,1} + p_1 p_{1,1} p_{1,0} p_{0,0} + p_1 p_{1,0} p_{0,1} p_{1,0} + p_1 p_{1,0} p_{0,0} p_{0,1} \\ &= 7\left(\frac{1}{2}\right)^4 = \frac{7}{16} \end{aligned}$$

**Système II : de la même manière et comme* $card(M') = 9$ *alors*

$$R_{II}(p) = P(\Phi_{II}(X) = 1) = P(X \in M') = 9\left(\frac{1}{2}\right)^4 = \frac{9}{16}........(\mathbf{0.75\ points})$$