** On a* $P_{1,1}^{(n)} = 0 \; \forall n \in \mathbb{N}^*$ *Alors* $\left\{ n \in \mathbb{N}^*, P_{1,1}^{(n)} \neq 0 \right\} = \Phi$ *donc* $d(1) = 0$**..................(0.5 points)**

** On a* $P_{2,2}^{(n)} = 0 \; \forall n \in \mathbb{N}^*$ *Alors* $\left\{ n \in \mathbb{N}^*, P_{2,2}^{(n)} \neq 0 \right\} = \Phi$ *donc* $d(2) = 0$**..................(0.5 points)**

** On a* $P_{3,3}^{(n)} = 0 \; \forall n \in \mathbb{N}^*$ *Alors* $\left\{ n \in \mathbb{N}^*, P_{3,3}^{(n)} \neq 0 \right\} = \Phi$ *donc* $d(3) = 0$**..................(0.5 points)**

** On a* $P_{4,4} = \frac{1}{4}$ *il y a une boucle donc* $d(4) = 1$**..................(0.5 points)**

** On a* $P_{5,5} = 1$ *il y a une boucle (5 état absorbant) donc* $d(5) = 1$**..................(0.5 points)**

5) Calculer les probabilités

* $P_{1,4}^{(5)} = P_{1,2}P_{2,3}P_{3,4}P_{4,4}P_{4,4} + P_{1,2}P_{2,4}P_{4,4}P_{4,4}P_{4,4} + P_{1,3}P_{3,4}P_{4,4}P_{4,4}P_{4,4} + P_{1,4}P_{4,4}P_{4,4}P_{4,4}P_{4,4} = \frac{1}{4}\frac{1}{2}\frac{3}{4}\frac{1}{4}\frac{1}{4} + \frac{1}{4}\frac{1}{2}\frac{1}{4}\frac{1}{4}\frac{1}{4} + \frac{1}{4}\frac{3}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4} + \frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4} = \frac{3}{256}$**........(0.5 points)**

* $P_{3,4}^{(2)} = P_{3,4}P_{4,4} = \frac{3}{4}\frac{1}{4} = \frac{3}{16}$**..................(0.25 points)**

* $P(X_2 = 3 \,|X_0 = 1) = P_{1,3}^{(2)} = P_{1,2}P_{2,3} = \frac{1}{4}\frac{1}{2} = \frac{1}{8}$**..................(0.25 points)**

* $P(X_4 = 4 \,|X_0 = 2) = P_{2,4}^{(4)} = P_{2,3}P_{3,4}P_{4,4}P_{4,4} + P_{2,4}P_{4,4}P_{4,4}P_{4,4} = \frac{1}{2}\frac{3}{4}\frac{1}{4}\frac{1}{4} + \frac{1}{2}\frac{1}{4}\frac{1}{4}\frac{1}{4} = \frac{1}{32}$**.........(0.5 points)**

* $P(X_4 = 4 \,|X_0 = 1) = P_{1,4}^{(4)} = P_{1,2}P_{2,3}P_{3,4}P_{4,4} + P_{1,2}P_{2,4}P_{4,4}P_{4,4} + P_{1,3}P_{3,4}P_{4,4}P_{4,4} + P_{1,4}P_{4,4}P_{4,4}P_{4,4} = \frac{1}{4}\frac{1}{2}\frac{3}{4}\frac{1}{4} + \frac{1}{4}\frac{1}{2}\frac{1}{4}\frac{1}{4} + \frac{1}{4}\frac{3}{4}\frac{1}{4}\frac{1}{4} + \frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4} = \frac{3}{64}$**.......(0.5 points)**

6) Donner la loi de X_n ***dans le cas ou*** X_{n-1} ***prend que les deux valeurs*** 1 ***et*** 2 ***avec même probabilité***

On a X_{n-1} *prend que les deux valeurs* 1 ***et*** 2 *avec même probabilité c. à. d :* $\mu_{n-1} = \left(\frac{1}{2}, \frac{1}{2}, 0, 0, 0\right)$. *Alors la loi* μ_n *de* X_n *est donnée par***........(1 points)**

$$\begin{aligned}
\mu_n &= \mu_{n-1}P \\
&= \left(\frac{1}{2}, \frac{1}{2}, 0, 0, 0\right) \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{3}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \\
&= \left(0, \frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}\right)
\end{aligned}$$