corrigé type (Analyse complexe)

Exercice 01 ; On a : $0 < a < b$, $C_r = \{ z : |z| = r, r > 0 \}$

si $r < a$, $\Rightarrow \displaystyle\int_{C_r} \frac{dz}{(z-a)(z-b)} = 0$ (1,5)

si $a < r < b$ $\Rightarrow \displaystyle\int_{C_r} \frac{dz}{(z-a)(z-b)} = \int_{C_r} \frac{dz/(z-b)}{z-a} = 2\pi i\, g(a)$ / $g(z) = \dfrac{1}{z-b}$

$= \dfrac{2\pi i}{a-b}$ (1,5)

* si $b < r$ $\Rightarrow \displaystyle\int_{C_r} \frac{dz}{(z-a)(z-b)} = \frac{1}{a-b}\int_{C_r} \frac{dz}{z-a} + \frac{1}{b-a}\int_{C_r} \frac{dz}{z-a}$

$= \dfrac{2\pi i}{a-b} + \dfrac{2\pi i}{b-a} = 0$. (1,5)

Exercice 02 : $f(z) = \dfrac{z^2 + e^{iz}}{z(z-2i)^2} + \dfrac{1}{(z+3)^3}$

sur Γ_1, on a $z_0 = 0 \in \Gamma_1$ donc

$\displaystyle\int_{\Gamma_1} f(z)\,dz = \int_{\Gamma_1} \frac{(z^2 + e^{iz})/(z-2i)^2}{z}\,dz = 2\pi i\, g(0)$ / $g(z) = \dfrac{z^2 + e^{iz}}{(z-2i)^2}$

$= \boxed{\dfrac{-\pi i}{2}}$ (1,5)

* sur Γ_2, on a $z_0 = 2i \in \Gamma_2$, donc :

$\displaystyle\int_{\Gamma_2} f(z)\,dz = \int_{\Gamma_2} \frac{(z^2 + e^{iz})/z}{(z-2i)^2} = 2\pi i\, g'(2i)$ / $g(z) = \dfrac{z^2 + e^{iz}}{z}$

$= 2\pi i\left(1 + \frac{3}{4}e^{-2}\right)$

$= \boxed{2\pi i + \dfrac{3\pi}{2} e^{-2}}$ (1,5)

sur Γ_3, on a $z_0 = -3 \in \Gamma_3$, donc

$\displaystyle\int_{\Gamma_3} f(z)\,dz = \int_{\Gamma_3} \frac{dz}{(z+3)^3} = \frac{2\pi i}{2!} g''(-3) = 0$ (1) / $g(z) = 1 \Rightarrow g'(z) = 0$

Γ_4 : f est holomorphe sur Γ_4 donc $\displaystyle\int_{\Gamma_4} f(z)\,dz = 0$ (1)

$\displaystyle\int_{\Gamma} f(z)\,dz = \int_{\Gamma_1} f(z)\,dz + \int_{\Gamma_2} f(z)\,dz + \int_{\Gamma_3} f(z)\,dz + \int_{\Gamma_4} f(z)\,dz$

$= -\dfrac{\pi i}{2} + 2\pi i + \dfrac{3\pi}{2} e^{-2} = \boxed{\dfrac{3\pi}{2}\left(1 + e^{-2}\right)}$ (2)