Oum El Bouaghi University
Department of Mathematics and Informatics
Master 1: Mathematics
Optimization under constraints
Final Exam solution
21-05-2023

Exercise 01: (See the course)

-(*01 POINT*) The formula of constrained optimization problem in general case under equality constraints.

-(*01 POINT*)Lagrange's theorem

-(*03 POINTS*) and theorem (take $p = 1$ and $n = 2$).

Exercise 02:

-(*01.5 POINTS*) Feasible set is compact (bounded and closed)

-(*01.5 POINTS*) Lagrange conditions are satisfied (write the theorem of Lagrange) .

-(*01 POINTS*) The critical points of $f(x, y)$

$$L(x, y) = ln(x - y) - \blacksquare(x2 + y2 - 2)$$

$$\nabla L(x, y) = 0$$

$$\begin{cases} \frac{1}{x-y} - 2\lambda x = 0 \\ -\frac{1}{x-y} - 2\lambda y = 0 \end{cases}$$

(*02 POINTS*)Finaly, we can find 4 points (rejected 3 points and accepted one point) because $x > y$

$$x = (1, -1)$$

-(*01.5 POINT*) Determine the optimal solution of $f(x, y)$.

the function $f = ln$ is concave function and -g(x,y) is also concave so the sum of two concaves is concave function then the point $x = (1, -1)$ present the maximum of $f(x, y)$.

Exercise 03:

$$\begin{cases} \min f(X) = \min - \sum_{i=1}^{n} x_i^2 \\ st \\ \Omega = \left\{ (x_1, ...x_n) \in \mathbb{R}^n, \varphi(x_1, ...x_n) = \sum_{i=1}^{n} x_i^4 - 1 \leq 0 \right\} \end{cases}$$

-(*01 POINT*) The existence of the optimum (compact set)
-(*01 POINT*) Constraints are qualified
-(*01.5 POINT*) Using **KKT** formula to solve the problem
(*01.5 POINT*)By the formula of KKT we find
If $\lambda = 0$ we have $x = 0$
If $\lambda \neq 0$ we have

$$-2(x_1, x_2, ..., x_n) + 4\lambda(x_1^3, x_2^3, ..., x_n^3) = 0$$

$$\forall i \in [1, n] - 4\lambda x_i \left(\frac{1}{2\lambda} - x_i^2\right) = 0$$

So,

$$\begin{cases} x_i = 0 \\ or \\ \frac{1}{2\lambda} = x_i^2 \end{cases}$$

Then

$$\frac{n_0}{4\lambda^2} = 1$$

with n_0 is the number of x such that $x_i \neq 0$
-(*02.5 POINT*) Give the vector X^*such that $f(X^*) = \min_{\Omega} f(X)$
f admet a minimum under the form

$$\frac{1}{\sqrt{2\sqrt{n}}}(\pm 1, \pm 1, ..., \pm 1)$$