

e) à basse température ($T \ll \theta_D \rightarrow n \gg 1$), $n_D \rightarrow \infty$

d'où:

$$\int_0^{n_D} \rightarrow \int_0^{\infty}$$

(015)

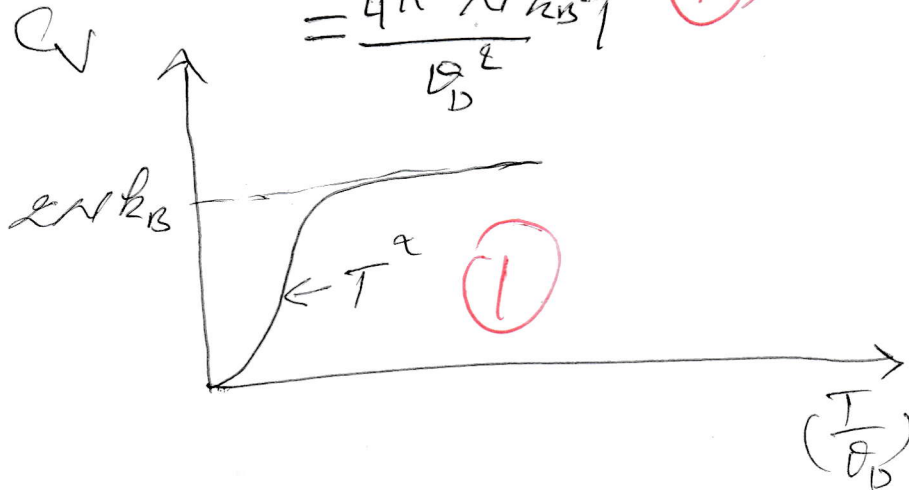
$$U(T) = \frac{4N}{\beta^3 h^3 \omega_D^3} \int_0^{\infty} \frac{n^2 dn}{e^n - 1} \approx \frac{\pi^2}{3}$$

Alors: $U(T) = \frac{4N}{\beta^3 h^3 \omega_D^3} \times \frac{\pi^2}{3} = \frac{4N}{3} \times \frac{4N k_B T^3}{\beta^3 \omega_D^3}$

$$U(T) = \frac{\pi^2}{3} \times \frac{4N k_B T^3}{\theta_D^3} \quad (015)$$

et: $C_V = \frac{\partial U(T)}{\partial T} = \frac{2}{3} \times \frac{4N k_B T^2}{\theta_D^3} \rightarrow C_V \propto T^2$

$$= \frac{4\pi^2 N k_B T^2}{\theta_D^3} \quad (015)$$



(3)