3) on a : $U = \int_0^{\omega_D} \langle E_n \rangle D(\omega) d\omega$ 0,5

$\langle E_n \rangle = (\langle n_{ph} \rangle + \frac{1}{2}) \hbar\omega$ / $\langle n_{ph} \rangle = \frac{1}{e^{\beta\hbar\omega} - 1}$

Alors : $U = \int_0^{\omega_D} \frac{\hbar\omega D(\omega) d\omega}{e^{\beta\hbar\omega} - 1} + \frac{1}{2}\int_0^{\omega_D} \hbar\omega D(\omega) d\omega$

$$U(T) = \int_0^{\omega_D} \frac{\hbar\omega D(\omega) d\omega}{e^{\beta\hbar\omega} - 1}$$

$$\boxed{U(T) = \frac{4N}{\omega_D^2} \int_0^{\omega_D} \frac{\hbar\omega^2 d\omega}{e^{\beta\hbar\omega} - 1}}$$ 0,5 / $$\boxed{C_V(T) = \frac{\partial U(T)}{\partial T}}$$

On pose : $x = \beta\hbar\omega$, 0,5 $x_D = \beta\hbar\omega_D = \frac{\theta_D}{T}$

$dx = \beta\hbar d\omega$, $\beta = \frac{1}{k_B T}$

$\hbar\omega_D = k_B\theta_D$

$$U(T) = \frac{4N}{\omega_D^2} \int_0^{x_D} \frac{\cancel{x}^2 dx}{\beta^3\hbar^2(e^x - 1)} = \boxed{\frac{4N}{\beta^3\hbar^2\omega_D^2} \int_0^{x_D} \frac{x^2 dx}{e^x - 1}}$$

$C_V = ?$ / à ~~haute~~ haute température : $T \gg \theta_D \rightarrow x \ll 1$

0,5

$e^x \simeq 1 + x$; Alors, $U(T) = \frac{4N}{\beta^3\hbar^2\omega_D} \int_0^{x_D} \frac{x^2 dx}{x}$

$$U(T) = \frac{4N}{\beta^3\hbar^2\omega_D^2} \int_0^{x_D} x\, dx = \frac{2N x_D^2}{\beta^3\hbar^2\omega_D^2} = \boxed{2Nk_B T}$$

d'où : $C_V = \frac{\partial U(T)}{\partial T} = \boxed{2Nk_B}$ 0,5

2