

Oum El Bouaghi University
 Department of Mathematics and Informatics
 Master 1: Mathematics
 Optimization under constraints
 Final Exam
 21-05-2023

Exercise 01:

- Give the formula of constrained optimization problem in general case under equality constraints.
- Write Lagrange's theorem corresponding to the problem and prove it (take $p = 1$ and $n = 2$).

Exercise 02:

We formulate the problem as the following function $f(x)$ under the constraint $g(x, y) = 0$ (equality constraint).

$$f(x, y) = \ln(x - y), \quad g(x, y) = x^2 + y^2 - 2.$$

- Study the compactness of feasible set.
- Verify that Lagrange conditions are satisfied (with $x > y$).
- Find all critical points of $f(x, y)$ under the constraint $g(x, y)$ by Lagrange's method
- Determine the optimal solution of $f(x, y)$.

Exercise 03:

Consider the following problem

$$\left\{ \begin{array}{l} \min f(X) = \min - \sum_{i=1}^n x_i^2 \\ \text{st} \\ \Omega = \left\{ (x_1, \dots, x_n) \in \mathbb{R}^n, \varphi(x_1, \dots, x_n) = \sum_{i=1}^n x_i^4 - 1 \leq 0 \right\} \end{array} \right.$$

- Study the existence of the optimum solution
- Prove the qualification of constraints
- Using **KKT** formula to solve the problem
- Give the vector X^* such that $f(X^*) = \min_{\Omega} f(X)$