Corrige -type

Exercice 01

$$N_1 = 45. \quad \bar{x}_1 = 51{,}80. \quad \sigma_{e_1} = 3{,}82\,. \qquad N_2 = 40. \quad \bar{x}_1 = 55{,}2. \quad \sigma_{e_2} = 4{,}02\,.$$

1. $\begin{cases} H_0: \mu_1 = \mu_2 \\ H_1: \mu_1 \neq \mu_2 \end{cases}$

2. $\alpha = 0{,}05$.

3. Test T : Comparaison entre deux moyennes

4. $\sigma_1$et σ_2 sont inconnus, $N_1, N_2 \geq 30$ et $\sigma_1 = \sigma_2$

$$\varepsilon_{cal} = \frac{|\,\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{(\hat{\sigma}_{e_1})^2}{N_1} + \frac{(\hat{\sigma}_{e_2})^2}{N_2}}}$$

$$\hat{\sigma}_{e_1} = \sqrt{\frac{N_1}{N_1-1}} \times \sigma_{e_1} = \sqrt{\frac{45}{44}} \times 3{,}82 = 3{,}85 \Longrightarrow (\hat{\sigma}_{e_1})^2 = 14{,}88.$$

$$\hat{\sigma}_{e_2} = \sqrt{\frac{N_1}{N_1-1}} \times \sigma_{e_2} = \sqrt{\frac{40}{39}} \times 4{,}02 = 4{,}06 \Longrightarrow (\hat{\sigma}_{e_2})^2 = 16{,}48.$$

$$\varepsilon_{cal} = \frac{|51{,}80 - 55{,}2|}{\sqrt{\frac{14{,}88}{45} + \frac{16{,}48}{40}}} = 3{,}95.$$

5. $\varepsilon_{cal} > \varepsilon_{0{,}05} = 1{,}96 \Longrightarrow$ H_0 est rejetée.

Exercice 02

12	14,5	11,9	12,4	15	13,1	11,8	15,2

1. Calculs : Moyenne et Variance

- $$\bar{x} = \frac{1}{N}\sum n_i x_i = \frac{1}{8}(105{,}9) = 13{,}23.$$

- $$v_x = \left[\frac{1}{8}\left(\sum (x_i)^2\right)\right] - (\bar{x})^2 = \left[\frac{1}{8}(1416{,}51)\right] - (7{,}55)^2 = 13{,}23.$$

  $$\sigma_e = \sqrt{v_x} = \sqrt{2{,}03} = 1{,}42.$$

- $$c.v = \frac{\sigma_e}{\bar{x}} \times 100\% = \frac{1{,}42}{13{,}23} \times 100\% = 10{,}73\% < 20\%.$$

L'échantillon est homogène.

2. Intervalle de confiance : Moyenne

σ inconnu et $N = 8 < 30$

$$Ic = \left[\bar{x} - T_{(\alpha;N-1)} \frac{\hat{\sigma}}{\sqrt{N}} ; \bar{x} + T_{(\alpha;N-1)} \frac{\hat{\sigma}}{\sqrt{N}}\right]$$

$$\hat{\sigma} = \sqrt{\frac{N}{N-1}} \times \sigma_e = \sqrt{\frac{8}{7}} \times 1{,}42 = 1{,}5.$$

$$Ic = \left[13{,}23 - 3{,}449 \frac{1{,}5}{\sqrt{8}} ; 13{,}23 + 3{,}449 \frac{1{,}5}{\sqrt{8}}\right]$$

$$= [13{,}23 - 1{,}84; 13{,}23 - 1{,}84]$$

$$= [6{,}39; 8{,}71].$$

3. Intervalle de confiance : Variance

μ inconnu

$$Ic = \left[\frac{\sum(x_i - \bar{x})^2}{\chi^2_{1-\frac{\alpha}{2}}(N-1)}, \frac{\sum(x_i - \bar{x})^2}{\chi^2_{\frac{\alpha}{2}}(N-1)}\right]$$

$$\sum(x_i - \bar{x})^2 = N \times (\sigma_e)^2 = 8 \times (1{,}42)^2 = 16{,}13.$$

$$Ic = \left[\frac{16{,}13}{16{,}01}, \frac{16{,}13}{1{,}69}\right] = [1{,}00; 9{,}54].$$

Exercice 03

Sol. 01	9,5	8,6	9,4	7,4	7	6,5	5,9
Sol. 02	7	5,8	8,4	6	6,5	7,1	
Sol. 03	9,1	8,2	6	8,8	7	6,1	7,4

$N_1 = 7.$ $N_2 = 6.$ $N_3 = 7.$ $K = 2\,.$

$$S_1 = \sum x_i = 9{,}5 + 6{,}8 + 9{,}4 + 7{,}4 + 7 + 6{,}5 + 5{,}9 = 54{,}3.$$

$$S_2 = \sum x_i = 7 + 5{,}8 + 8{,}4 + 6 + 6{,}5 + 7{,}1 = 40{,}8.$$

$$S_3 = \sum x_i = 9{,}1 + 8{,}2 + 6 + 8{,}8 + 7 + 6{,}1 + 7{,}4 = 52{,}6.$$

$$S_1^2 = \sum x_i^2 = (9{,}5)^2 + (6{,}8)^2 + (9{,}4)^2 + (7{,}4)^2 + (7)^2 + (6{,}5)^2 + (5{,}9)^2 = 433{,}39.$$

$$S_2^2 = \sum x_i^2 = (7)^2 + (5{,}8)^2 + (8{,}4)^2 + (6)^2 + (6{,}5)^2 + (7{,}1)^2 = 281{,}86.$$

$$S_2^3 = \sum x_i^2 = (9{,}1)^2 + (8{,}2)^2 + (6)^2 + (8{,}8)^2 + (7)^2 + (6{,}1)^2 + (7{,}4)^2 = 404{,}46.$$

1. $\begin{cases} H_0 : \mu_1 = \mu_2 = \mu_3 \\ H_1 : \text{Les } \mu_i \text{ ne sont pas tous égaux} \end{cases}$

2. $\alpha = 0{,}05$.

3. Test : Analyse de la variance à un facteur (Anova 01).

4. Calcul de la statistique de test

$$S = S_1^2 + S_2^2 + S_3^2 - \frac{(S_1 + S_2 + S_3)^2}{N} = 1119{,}71 - 1090{,}76 = 28{,}95.$$

$$B = \frac{[54{,}3]^2}{7} + \frac{[40{,}8]^2}{6} + \frac{[52{,}6]^2}{7} - \frac{(54{,}3 + 40{,}8 + 52{,}6)^2}{20} = 1093{,}90 - 1090{,}76 = 3{,}13.$$

$$W = S - B = 28{,}95 - 3{,}13 = 25{,}82.$$

5. Tableau de variance

S.V	S.S	D.f	S.M	F_{Cal}
	$B = 3{,}13$	$k - 1 = 3$	$R = \frac{B}{k-1} = 1{,}56$	$F = \frac{R}{H} = 1{,}03$
	$W = 25{,}82$	$N - k = 17$	$H = \frac{W}{N-k} = 1{,}51$	
	$S = 28{,}95$	$N - 1 = 19$		

On a $F = 1{,}03 < F_{(0{,}05;17;2)} = 3{,}592 \Longrightarrow$ H_0 est acceptée.