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Application of Markovian Models and Transition Probabilities' Matrix To Analyze the Workforce Movement in Jordanian Productivity Companies

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Abstract :

This study aimed to apply of the Markovian models and transition probabilities' matrix to analyze the movement of the workforce in Jordanian Productivity Companies. To achieve the study objectives, the study is mainly based on the secondary data related to workforce movement selected from the annual reports of the Jordanian productivity companies (potash, phosphate, and pharmaceutical) for year (2014). The study reached to a number of results, it was the most important:

a. The estimation of the Transition Probabilities, refers to the highest retention percentage was in favor of the Phosphate company, followed by Potash company, and finally followed by the Pharmaceutical company.

b. The estimation of the Transition Probabilities, refers to the lowest loss was in favor of the Phosphate company, for Potash company and Pharmaceutical company.

Upon the foregoing results, the study reached to a number of conclusions, recommendations.

Keywords: Marcov Analysis, Transition Probabilities**,** Stochastic Processes, Labor Turnover, Workforce Planning**.**

الملخّص:

هدفت هذه الدراسة إلى تطبيق نماذج ماركوف ومصفوفة الاحتمالات الانتقالية لغرض تحليل حركة القوى العاملة في الشركات الإنتاجية الأردنية. ولتحقيق أهداف الدراسة، فقد استندت بشكل رئيسي على البيانات الثانوية المتعلقة بحركة القوى العاملة اختيرت من التقارير السنوية للشركات الإنتاجية الأردنية (البوتاس، والفوسفات، والأدوية) لسنة (2014). وتوصلت الدراسة إلى عدد من النتائج، كان من أهمها:

أ. أشارت نتائج تقدير الاحتمالات الانتقالية، إلى أعلى نسبة احتفاظ كانت لصالح شركة الفوسفات، تليها شركة البوتاس، وأخيرا تليها شركة الأدوية.

ب .أشارت نتائج تقدير الاحتمالات الانتقالية، إلى أوطأ خسارة كانت لصالح شركة الفوسفات، إلى كل من شركة البوتاس وشركة الدوائية.

وبناء على النتائج السابقة، توصلت الدراسة إلى عدد من الاستنتاجات والتوصيات.

الكلمات المفتاحية: تحليل ماركوف، الاحتمالات الانتقالية، العمليات العشوائية، دوران العمل، تخطيط القوى العاملة.

1. Introduction

The workforce planning on the basis of established process requires a good knowledge of those deployed in the establishment, as well as entry and dropout and promotion of employees in order to reach a future plan fit and desire administration in determining the future policies of the workforce system. As the number of employees at the facility is fixed where noted that this number will increase to the gallery by the entry of new employees of the establishment or system to represent (gain for establishment), or exposed to decrease due to the leakage of workers outside the entity to represent the (loss for establishment). As a result of this movement of workers, and the movement of entering and there leakage. To achieve that can be used **Markov Analysis**, which is one of the easiest methods to describe a movement of the employees and thus predict the numbers of employees within the enterprise, and using the **Transition Probabilities' Matrix** that seem more accurate in the workforce planning.

2. Methodology

2.1. The Study Problem

The study problem revolves around the existence of many productivity or service organizations were suffered from a problem of the labor turnover, and which would inevitably affect on the policies, plans, and institutional performance. In order to deal this problem should be on the administrations of these organizations use the scientific methods and mathematical models for human resources planning and workforce movement in the organization.

In the light of reportedly, Markov models are considered appropriate models for these problems, and to predict the workforce that you need these organizations in the subsequent periods.

2.2. The Study Objects

The objectives of this study can be summarized as follows:

a. To identify the concept of Markovian Analysis and some models related to Markov chains.

b. Finding the transition possibilities' matrix for adoption to analyze the workforce movement, and used it to forecast in market shares for productivity organizations.

c. To identify the concept of workforce planning for the organizations, and determine the importance of that in dealing with the problem of labor turnover.

d. Forecasting in the organizations workforce for the years (2015 and 2016), and the number workforce in period of the Market Equilibrium**.**

e. Offer some conclusions and recommendations to the decision makers in the Jordanian productivity companies.

3. The Theoretical Part & Literature Review

3.1. The Workforce Planning Concept

Has become a study of workforce and planning of important topics, to the importance enjoyed by the workforce as a factor leading producer and engine of the natural resources on the one hand, and the impact and the importance of workforce planning in achieving economic development as a whole on the other hand the process.

As a result of the development in the economic, technological and social field, had the subject of workforce planning and widely watched by politicians, administrators and economists, and because of the demonstrated this development of a significant shortfall workforce resulting in a planning consideration of the workforce a props and basic pillars of economic development plans, where you specify the dimensions this process in the light of the mobilization of manpower and appropriate efficient distribution and use of the various economic sectors, which collectively contribute to the economic development process.

In light of the foregoing, the aim of workforce planning is to meet the future requirements of the labor force or what is available in the future in light of the multiple objectives such as economic conditions, governmental and instructions as well as the history of the facility and its policies regarding recruitment, promotion and dropout.

3.2. The Stochastic Processes

The random variables that are generated by probabilistic laws set had defined (stochastic process), Lawer has pointed to such operations as random changed by the time variable and is denoted by (X_t) and rely upon mathematical laws, and symbolizes the stochastic process symbol (X_n), such that (n) refers to the discrete time (n = 0,1,2…).

The values which assumed by stochastic process (X_t) called (States) and the set of values called the (State space) and the range of possible values for the parameter marked Index parameter called the (Parameter Space) may be discrete or continuous and parameter (T) usually refers to time.

3.3. Markov analysis

Markov analysis is a well-known quantitative methods, whereby the analysis of the current changes to the phenomenon that, in order to predict future changes to this phenomenon, this method usually falls within the Operations Research.

Credited Markov analysis method to the Russian world (A. Markov) (1922- 1952), was limited to use this method in the first instance on the physical applications to study the movement of gas molecules in a closed vessel in order to predict the movement of these molecules in the future.

The Markov processes is a special case of stochastic or random processes, and look for these operations as a series of situations experienced by the phenomenon through a certain period of time or processes through which a moving object through different periods of time series, called the mentioned operations series by (Markov Chain). In order to identify the Markov processes must be used mathematical analytical methods and conclusions and to clarify the distinctive properties during the development process.

Based on the foregoing, the Markov analysis defined as "Mathematical and scientific method to analysis of the behavior of different phenomena's during the current period in order to predict the behavior of these phenomena's in the future in any later periods" (Touama, 2009: 297).

3.3.1. Assumptions of Markov Analysis

An analysis of the Markov method is based on the fundamental assumption that: (any system is dealt with in the first instance be in its initial state, in preparation for the transition to another state), and this assumption based on a certain probability laws called the (Transition Probabilities), which are known as a "transition probabilities of a particular case to another case during a certain period of time".

For example, the probability of transition phenomenon of the case (i) in the current period (n) to another state (j) in the later period (n+1) writes as follows (Touama, 2009: 298-299):

$$P\{X_{n+1} = j / X_n = i\} = P_{ij} \quad , \quad \forall\, i , j \qquad \ldots(1)$$

Whereas:

X_n: Value of the phenomenon in the current period (n).

X_{n+1}: Value of the phenomenon in the subsequent period (n+1).

P_{ij}: Probability of transmission the phenomenon of state (i) to state (j).

3.3.2. Transition Probabilities' Matrix

The traditional status the transition probabilities (P_{ij}) values putting in a square matrix $P = [\ P_{ij}\ ]$, which takes the following form:

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \ldots P_{1n} \\ P_{21} & P_{22} & P_{23} & \ldots P_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ P_{m1} & P_{m2} & P_{m3} & \ldots P_{mn} \end{bmatrix} \qquad \ldots(2)$$

The above matrix called the (Transition Probabilities' Matrix), or sometimes called (Markov Matrix), which represents the matrix of the stochastic or random processes, in which the sum of the probabilities of any row ranks equal to the one, that is:

$$\sum_{j=1}^{n} P_{ij} = 1 \quad , i = 1,2,\ldots,m \qquad \ldots(3)$$

The elements (Pij) of transition probabilities' matrix representing the transition probability from the state (i) to the state (j) by one-step or one time period, if we want to find the probability value of the movement of phenomenon from the state (i) to the state (j) with a limited number of steps or time periods of (m), so the element P_{ij}^m can be written as follows (Touama, 2009: 300):

$$P_{ij}^m = P\{ X_{n+m} = j / X_n = i\} \quad , \forall\, i , j \;\&\; (n, m \in N) \qquad \ldots(4)$$

In order to obtained the Transition Possibilities' matrix, we find the following:

a. Ratio of Customers' Retention

It means that the organization's ability to retain with a greater percentage of it's customers, and it calculated according to the following formula:

$$P_{ii} = 1- L_i / V_i \qquad \ldots(5)$$

Whereas:

P_{ii}: Retention probability by the customers for the organization (i), and these probabilities are located on the diagonal of the matrix .

L_i : The customers number who are losing them the organization (i).

V_i : The customers number at the beginning period for the organization (i).

b. The loss Probability of Customers

It means the loss probability of the organization to it's customers to another organizations, and it calculated according to the following formula:

$$P_{ij} = N / V_i \quad , \quad j = 1,2,\ldots,n \qquad \ldots(6)$$

Whereas:

P_{ij} : Loss probability of the organization (i) to the organization (j), and these probabilities are located outside the diagonal of the matrix.

N : The customers number who are losing them organization (i) to another organizations.

3.3.3. Forecasting in Market Shares for Organizations

We can determine the share of each organization in the **current period**, according to the following formula (Touama, 2009: 304):

$$Share_{(i)} = V_i / V \qquad \ldots(7)$$

Whereas:

$Share_{(i)}$ **:** Market share of the organization (i) in the current period**.**

V : Sum of customers at the beginning period, such that; $[V = \sum_{i=1}^{m} V_i]$.

After that we can forecast the Market shares of organizations for subsequent periods, by the following equation:

$$\underline{S}\, P = \underline{S} \qquad \ldots(8)$$

3.3.4. Conditions of the Market Equilibrium

The state of the market equilibrium, defined as "a situation in which the market shares of the organizations involved in the competition process in the market become steady-state without these shares are unchanged in Later periods" and can reach a state of the market equilibrium, through the following relations:

a. Assume that we have the row probability vector ($\underline{S}$), representing the market shares to (m) organizations, this means that (Touama, 2009: 307-309):

$$\underline{S} = [S_1, S_2, \dots, , S_m] \quad \dots(9)$$

Under the following constraint:

$$\sum_{i=1}^{m} S_i = 1 \quad \dots(10)$$

b. Assume that we have the **Forecasting of Market Shares** M**atrix** P = [P_{ij}], which is square matrix with degree (mxn).

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \dots P_{1n} \\ P_{21} & P_{22} & P_{23} & \dots P_{2n} \\ \dots & \dots & \dots & \dots \\ P_{m1} & P_{m2} & P_{m3} & \dots P_{mn} \end{bmatrix} \quad \dots(11)$$

And re-use the previous relations and solution its, can be accessed to the state of the market equilibrium, as follows:

$$S_1 + S_2 + S_3 + \dots + S_m = 1 \quad \dots(12)$$

$$\underline{S}\,P = \underline{S} \quad \dots(13)$$

After simplified the two relationships in (11) and (12), we get on the following equations:

$$S_1 + S_2 + S_3 + \dots + S_m = 1 \quad \dots(14)$$

$$\left.\begin{array}{l} S_1 P_{11} + S_2 P_{21} + \dots + S_m P_{m1} = S_1 \\ S_1 P_{12} + S_2 P_{22} + \dots + S_m P_{m2} = S_2 \\ \dots\dots\dots\dots \\ S_1 P_{1n} + S_2 P_{2n} + \dots + S_m P_{mn} = Sm \end{array}\right] \quad \dots(15)$$

It is clear from the equations in the above relationships (13) and (14), the presence (m+1) equation, and containing (m) of unknowns, and for the purpose of solving the previous linear equations system, should delete one of these equations for getting (m) of equations equal to the variables number (m).

After solving the linear equations system of (m) equation by using the deletion method, we will get the row probabilistic vector (S^*), and this vector represents the conditions of the market equilibrium, and the values of the row probabilistic vector indicate to the organizations' shares in period of the market equilibrium.

c. Determine the customers number in period of the market equilibrium, by using the following form:

$$\text{Customers Number in period of the Market Equilibrium} = \left(\sum_{i=1}^{m} V_i\right) * S_i\,\% \quad \ldots(16)$$

3.4. Literature Review

After taking a look at some studies related to Markovian Models and Transition Probabilities Matrix, a group of studies relevant to the study theme had been chosen. Al-Bayaa (2003) explained the difference between registry errors in Markov chains. Also, Alwan (2007), discussed the application of Markov chains in the field of communications using the simulation method. Al-Saadi (2007), applying the maximum likelihood and least squares methods to estimate the transition probabilities' matrix, and has a comparison between the two methods to find the difference between them. Al-ali, et al., (2009) they focused on analysis of market movement using Markov chains. Also, Ben Ali (2009) refer to Markov chains in order to reduce the risks that threaten the economic institutions, whereas Markov analysis of mathematical and analytical way to help the institution or the decision-makers on the knowledge developed in the future and predict the risk that you may encounter. Hussein (2009) discussed the use of Markov matrix in estimating the time of staying of the student in the Faculty of Law at the University of Damascus, and determine the number of graduates and forecasting of graduates numbers for 2010.

Some authors as (Henry N., 1971, Lee T.C. & Judge G.G., 1972, Collins L., 1977, Nain R.B. and Kanwar S., 1980, Jong P., and Makiom G., 1984, Keilson J., and Graduate S.,1998, As-suaidi A.H., 2002, and Al-Zaiadi S., 2003) indicated to the importance of using Markov analysis and the Transition Probabilities to prediction.

4. The Applied Part

4.1. Collection Data

The study is mainly depend on the secondary data related to workforce movement selected from the annual reports of the Jordanian productivity companies (potash, phosphate, and pharmaceutical) for year (2014). To achieve the study objects, the researcher select three companies from the Jordanian productivity companies in order to choose number of the employees, loss, and win of employees. As shown in Table (1) the following:

Table 1. The employees movement between companies being studied, and the employees number during the year (2014)

Companies	Potash Company	Phosphate Company	Pharmaceutical Company	Loss (L_j)	No. of Employees
Potash Co.	0	38	33	71	2066
Phosphate Co.	26	0	43	69	4056
Pharmaceutical Co.	32	41	0	73	2040
Win (W_i)	58	79	76	213	8162

4.2. The Results and Discussion

4.2.1. Estimation the Transition Probabilities' Matrix

We can find the transition probabilities' matrix by using the relations (5) and (6), as follows:

$$P = \begin{bmatrix} 0.966 & 0.018 & 0.016 \\ 0.006 & 0.983 & 0.011 \\ 0.016 & 0.020 & 0.964 \end{bmatrix}$$

Evidenced from the results of the Transition Possibilities' Matrix the following:

a. The highest retention percentage was in favor of the **Phosphate** company amounting to (0.983), followed by **Potash** company with retention percentage of (0.966), and finally followed by the **Pharmaceutical** company with retention percentage of (0.964).

b. The lowest loss was in favor of the **Phosphate** company amounting to (0.017), for **Potash** company by (0.006) and **Pharmaceutical** company by (0.011).

4.2.2. Forecasting in Market Shares for Companies

Depending on the relations (7) and (8) we got the forecasting of the Market shares for the years (2015, and 2016) respectively. As shown in Table (2) the following:

Table 2. Forecasting of the Market shares for the years (2015, and 2016)

Years	Potash Co.	Phosphate Co.	Pharmaceutical Co.
2014	0.253	0.497	0.250
2015	0.251	0.498	0.251
2016	0.249	0.499	0.252

Depend on the above Market shares, we can **forecast** the employees number of companies for the years (2015, and 2016). As shown in Table (3) the following:

Table 3. Forecasting in the employees number for the years (2015, and 2016)

Years	Potash Co.	Phosphate Co.	Pharmaceutical Co.
2014	2066	4056	2040
2015	2049	4064	2049
2016	2032	4073	2057

4.2.3. Determine the Conditions of Market Equilibrium

After we solve the equation which explained in the relations (14) and (15), we got the **Row vector of shares** under the conditions of market equilibrium, as follows:

$\mathbf{S}^* = [0.250 \quad 0.499 \quad 0.251]$

Also, if we use the relations (16), can be determine the **employees number** at the period of the Market Equilibrium. As shown in Table (4) the following:

Table 4. The employees number at the period of the Market Equilibrium

Companies	Percentage of Shares (S_i %)	Total Number ($V = \sum V_i$)	Number of Employees
Potash Co.	25.0%		2041
Phosphate Co.	49.9%	8162	4073
Pharmaceutical Co.	25.1%		2048
Total	100%	-	8162

The results in Table (4), refers to the final result that the **Potash** company it has **lost (25)** employees, for **Phosphate** company by **(17)** employee and **Pharmaceutical** company by **(8)** employee.

5. Conclusions and Recommendations

5.1. Conclusions

This part includes the most important conclusions in light of the results, as follows:

a. The results of the Transition Probabilities estimation, refers to the highest retention percentage was in favor of the Phosphate company, followed by Potash company, and finally followed by the Pharmaceutical company.

b. The results of the Transition Probabilities estimation, indicates to the lowest loss was in favor of the Phosphate company by (0.017), for Potash company and Pharmaceutical company.

c. The results of the estimation the employees number at the period of the market equilibrium, indicate that the Potash Company it has lost (25) employees, for Phosphate company and Pharmaceutical company.

5.2. Recommendations

The study came out a set of recommendations including:

a. Necessarily improve the work conditions of employees in companies under the study, in order to treat the problem of labor turnover.

b. Necessarily use the Markov models and Transition Probabilities, in order to analyze the workforce movement and forecasting the employees number in other organizations.

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